FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2024
OPEN TEST – I
PAPER –2
TEST DATE: 04-02-2024

Time Allotted: 3 Hours **Maximum Marks: 180**

General Instructions:

- The test consists of total 51 questions.
- Each subject (PCM) has 17 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Three Sections**: **Section-A, Section-B & Section-C**.
  Section – A *(01 – 04, 18 – 21, 35 – 38)*: This section contains **TWELVE (12)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.
  Section – A *(05 –07, 22 – 24, 39 – 41)*: This section contains **NINE (09)** questions. Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
  Section – B *(08 – 13, 25 – 30, 42 – 47)*: This section contains **EIGHTEEN (18)** numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.
  Section – C *(14 –17, 31 – 34, 48 – 51)*: This section contains **SIX (06) paragraphs.** Based on each paragraph, there are **TWO (02)** questions of numerical answer type. The answer to each question is a **NUMERICAL VALUE (XXXXX.XX).** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

MARKING SCHEME

Section – A (Single Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the correct option is chosen.
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–1	In all other cases.

Section – A (One or More than One Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If only (all) the correct option(s) is (are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial marks	:	+2	If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–2	In all other cases.

Section – B: Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If ONLY the correct integer is entered;
Zero Marks	:	0	Question is unanswered;
Negative Marks	:	0	In all other cases.

Section – C: Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the correct integer is entered;
Zero Marks	:	0	Question is unanswered;
Negative Marks	:	0	In all other cases.

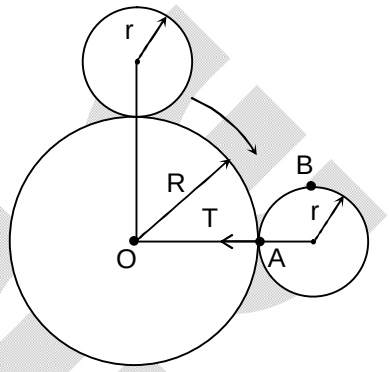
Physics

PART – I

SECTION – A
(One Options Correct Type)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

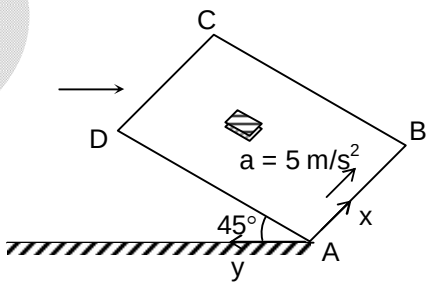
1. A circular disc is allowed to roll on the surface of a rough horizontal fixed cylinder of radius $R = \frac{9}{20}$m. The centres of the cylinder and the disc are connected by a massless rgid rod. The ends of the massless rigid rod are hinged at the centre of the disc and centre of the cylinder so that the disc can roll over the outer surface of the cylinder without slipping at any point during motion. The disc is released from rest from the top of the cylinder as shown in the figure. Radius of the disc is $r = \frac{3}{20}$m.



Hinged points are very smooth so that no loss of energy takes place. Consider the situation when the rod becomes horizontal the magnitude of acceleration of point A (which is on the disc and is in contact with the cylinder) is a_A and speed of point B is v_B. Then choose the correct option. (take g = 10 m/s^2)

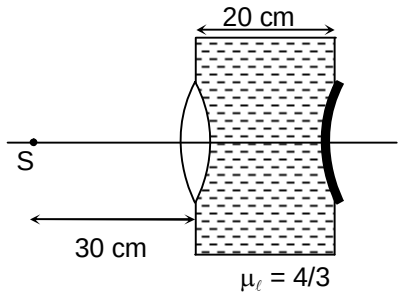
(A) a_A = 40 m/s^2 (B) a_A = 20 m/s^2
(C) v_B = 6 m/s (D) v_B = 8 m/s

2. An inclined plane ABCD is shown in the figure. It is inclined at an angle of 45° from horizontal. A block is placed on the inclined plane. The plane is accelerating along x-axis with an acceleration, a = 5 m/s^2. The minimum value of coefficient of friction between the block and the plane required, so that the block does not slip on the plane, is (g = 10 m/s^2)



(A) $\sqrt{1.5}$ (B) 1
(C) 2 (D) $\sqrt{2}$

3. A thin equiconvex lens of glass (refractive index μ_g = 3/2) having focal length 20 cm in air, is placed infront of a convex mirror of radius of curvature R_0 = 40 cm. The optical axis of the lens and the mirror are coinciding. The separation between the lens and the mirror is d = 20 cm and a transparent liquid (refractive index $\mu_i = \frac{4}{3}$) is filled between the lens and the mirror. A point source S of light is placed at a distance 30 cm from the lens and it is on the optical axis of the system as shown in the figure. Then, choose the correct option.



(A) Final image is formed at the soruce S itself
(B) Final image is formed at the pole of the mirror
(C) Final image is formed 10 cm behind the lens
(D) Final image is formed at infinity

4. Using dimensional analysis electrical resistivity can be expressed in terms of fundamental constant
$h \rightarrow$ Plank's constant
$m_e \rightarrow$ mass of electron
$e \rightarrow$ charge of electron
$c \rightarrow$ speed of light in vacuum and
$\varepsilon_0 \rightarrow$ permittivity of free space as

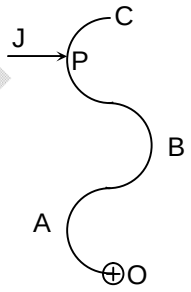
(A) $\frac{\varepsilon_0 m_e ce}{h}$ (B) $\frac{\varepsilon_0 m_e^2 ce^2}{h^2}$

(C) $\frac{\varepsilon_0 m_e^2}{ce^2}$ (D) $\frac{h^2}{m_e ce^2}$

SECTION – A
(One or More than one correct type)

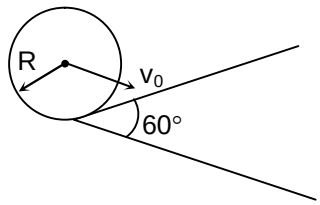
*This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

5. Three semicircular rings A, B and C of mass 5 kg, 10 kg and 5 kg respectively and equal radius 25 cm, are joined with each other. This structure is placed on a smooth horizontal surface and it is hinged at one of its ends such that it is free to rotate about a fixed vertical axis passing through the hinged end O without any friction. A horizontal impulse J = 48 N-s is applied at point P on it as shown in the figure. The correct statement(s) are



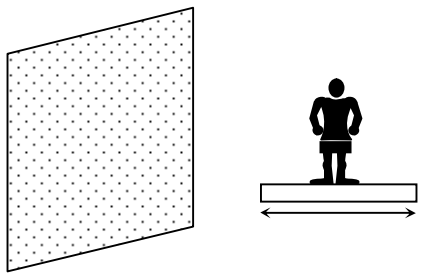
(A) The angular velcoity of the structure just after the impulse applied is 4 rad/s.
(B) The angular velcoity of the structure just after the impulse applied is 0.5 rad/s.
(C) The impulse applied by the hinged axis on the structure is 12 N-s.
(D) The impulse applied by the hinged axis on the structure is zero.

6. In the figure shown, two thin fixed rough horizontal rods rigidly joined at an angle $\theta = 60°$ are shown. A solid sphere of radius R = $\left(\frac{1}{13}\right)$ m is placed at joint of the rods and given a slight push so that it rolls with negligible speed on the structure of the rods symmetrically. (Assume friction is sufficiently large between the rods and the sphere to prevent slipping) (Take g = 10 m/s^2)



(A) Its angular speed increases and it becomes constant at 20 rad/s
(B) Its linear velocity first increases and then decreases in the horizontal direction
(C) Its angular speed increases continuously
(D) Torque on the sphere about a point on the joint of the rods is zero

7. A two dimensional very large and fixed source of sound is producing sound of frequency 3.3 KHz. Intensity of sound is I_0 = 100 W/m^2 as shown in the figure. An observer is oscillating infront of sound source along a horizontal line such that it always moves perpendicular to the plane of the source. Amplitude of oscillation is 5 m and time period is $\pi/2$ sec. Speed of sound in air is 330 m/s.



(A) Maximum frequency observed is 3.5 kHz.

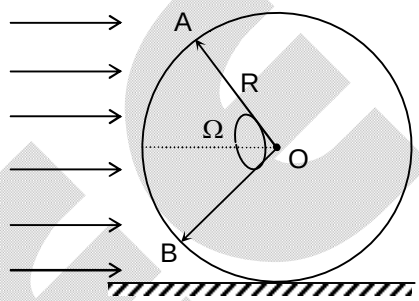
(B) Minimum frequency observed is 3 kHz
(C) Intensity of sound to the observer is constant
(D) Maximum intensity of sound to the observer is 106 W/m^2 (approx.)

SECTION – B

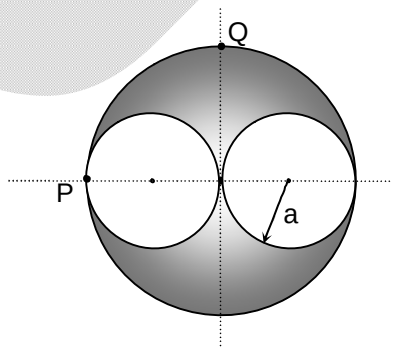
(Numerical Answer Type)

*This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.*

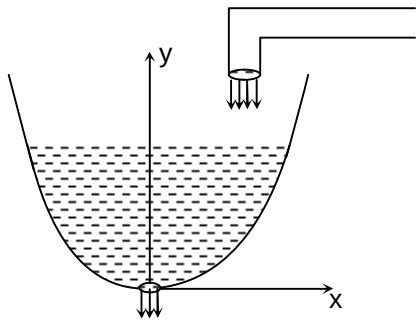
8. In the shown figure, a sphere of radius R is placed on a smooth and perfectly absorbing surface. Horizontal light of intensity I is falling on the sphere. The AB portion of the sphere making a solid angle $\Omega = \pi(2-\sqrt{2})$ Sr at centre O of the sphere is made perfectly absorbing (black) and the other portion of the surface is perfectly reflecting. Assuming there is no radiation is falling on the sphere from other sides. The force due to light on the sphere is $\frac{KI\pi R^2}{4c}$, where c is speed of light in free space. The value of k is



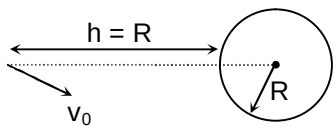
9. An infinitely long cylindrical conductor has radius 2a. It has two cylindrical cavities of radius a each. The cylinder has current flowing in it with uniform current density J. The ratio of the magnitudes of the magnetic field at Q and at P is $\frac{n}{5}$. The value of n is



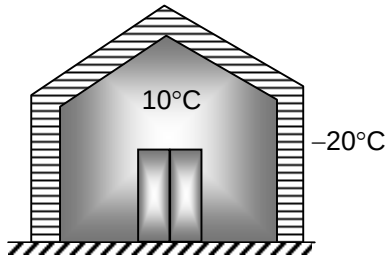
10. A bowl of the shape having parabolic ($y = kx^2$) cross section is being filled with a liquid with a pipe at a constant volume rate Q m^3/s. At the base of the bowl, a small hole of area a is present from where liquid is coming out of it. In the steady state, when the height of the liquid in the bowl becomes constant, the incoming liquid from the pipe is stopped. The time after which the height of the liquid becomes half, is $\frac{\pi Q^3}{Pka^4g^2}\left(1-\frac{1}{2\sqrt{2}}\right)$. The value of P is



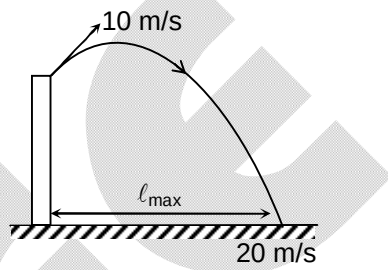
11. A particle is given some speed v_0 at a height h = R above the surface of earth such that it revolves around earth. The minimum value of speed v_0 required is $\sqrt{\frac{\alpha GM}{\beta R}}$. Find the value of ($\alpha$ + β). (Assume the mass of the earth is M and its radius is R)



12. Due to presence of the people in house, the inside of the house is maintained at a temperature 10°C where as its surrounding temperature is −20°C. When a room heater is switched on, the temperature of the room is increased and becomes steady at 15°C. When a light bulb of power 200 W is switched on, the temperature of the room is further increased by 1°C in the steady state. The power of the heater (in kilowatt) is



13. A particle is thrown from the top of a building with a speed 10 m/s and it reaches on the ground with a speed 20 m/s. Take acceleration due to gravity g = 10m/s^2 and viscous forces of air are negligible. The maximum distance from the foot of the building where it falls on the ground is 5K meter. Find the value of K.



SECTION – C
(Numerical Answer Type)

This section contains **TWO (02) paragraphs.** Based on each paragraph, there are **TWO (02)** questions of numerical answer type. The answer to each question is a **NUMERICAL VALUE (XXXXX.XX).** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

Paragraph for Question Nos. 14 and 15

1 kg of $_1H^2$ is used in fusion reactor to produce electric power.

The nuclear reaction is

$$_1H^2 +_1 H^2 \longrightarrow_2 He^4$$

It is given that mass of $_1H^2$ is 2.0141 u and mass of $_2He^4$ is 4.0026 u

The energy liberated is $P \times 10^{14}$ J

To convert this nuclear energy into electrical energy. The water at 300 K is heated to convert into steam at temperature 800 K. The left over steam is at 400 K. The energy supplied to electric grid is $Q \times 10^{14}$ J.

Specific heat capacity of water = 4.2×10^3 J/kg-k,

Specific heat capacity of steam = 2×10^3 J/kg-k,

Latent heat of vaporization of water $= 2.25 \times 10^6$ J/kg

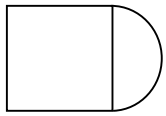
1amu = 931MeV/c^2

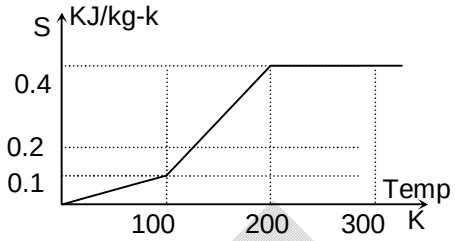
14. Find the value of P.

15. Find the value of Q.

Paragraph for Question Nos. 16 and 17

A solid cylinder and hemisphere are joined as shown in the figure. Combined mass of the cylinder and the hemisphere is $\frac{1}{3}$kg. The cylinder is painted shiny white and the hemisphere is painted dark black. The initial temperature of the body is T_i = 300 K and it is released form rest in gravity free space where the temperature of surrounding is 0K. The body attains terminal velocity v_0. Its specific heat capacity (S) is given in the shown graph.





16. Find the total energy (in KJ) radiated by the body.

17. Find the terminal velocity v_0 (in mm/s) of the body.

Chemistry

PART – II

SECTION – A
(One Options Correct Type)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

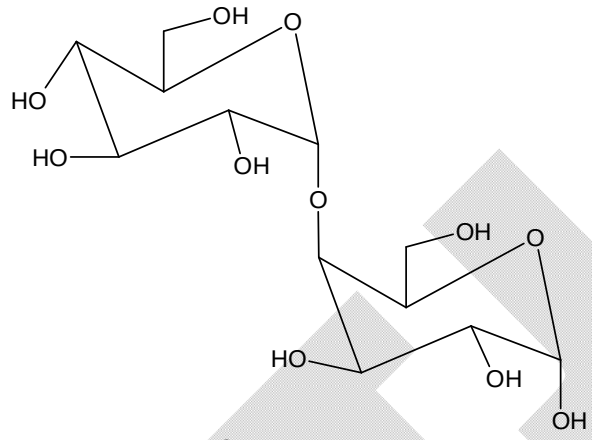
18. Which of the following is an example of a redox reaction?

(A) $XeF_4 + O_2F_2 \longrightarrow$ (B) $XeF_2 + PF_5 \longrightarrow$

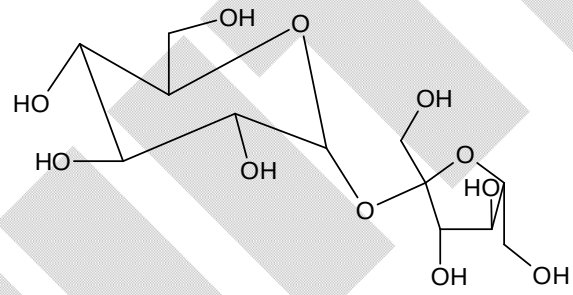
(C) $XeF_6 + H_2O \longrightarrow$ (D) $XeF_6 + 2H_2O \longrightarrow$

19. A disaccharide does NOT give a positive test for Tollen's reagent. Upon acidic hydrolysis, it gives an equimolar mixture of two different monosaccharides, both of which can be oxidized by bromine water. The correct structure of disaccharide is:

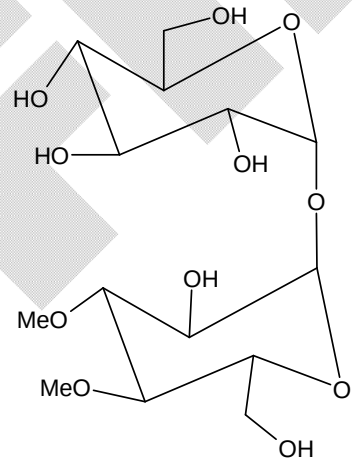
(A)



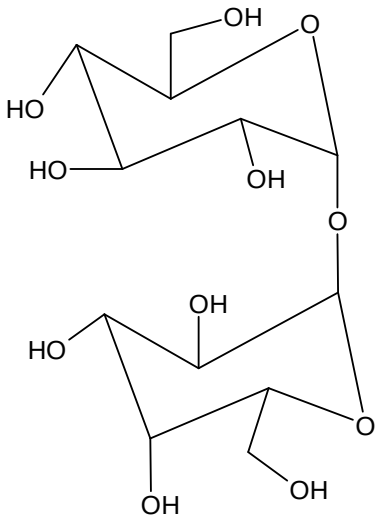
(B)



(C)



(D)



20. Which of the following statement is INCORRECT?
(A) In the metallurgly of aluminium, purified Al_2O_3 is mixed with Na_3AlF_6 and CaF_2 which lowers the melting point of the mixture and also increases the conductivity.
(B) Chalcocite and cinnabar are sulphide minerals.
(C) Self reduction technique is not possible during the metallurgy of mercury
(D) Leaching is the method to remove Fe_2O_3 quantitatively from red bauxite.

21. o-nitrotoluene is subjected to the following sequence of reactions:

$$\text{o}-\text{nitro toluene} \xrightarrow{\text{Sn/HCl}} (\text{A}) \xrightarrow[\text{anhydride}]{\text{acetic}} (\text{B}) \xrightarrow[\text{conc. } H_2SO_4]{\text{conc. } HNO_3} (\text{C}) \xrightarrow{\text{KOH(aq)}} (\text{D}) \xrightarrow[350^\circ\text{C}]{\text{Cl}_2\text{(1eqv.)}} (\text{E}) \xrightarrow[\text{2. Cu/HCl/}\Delta]{\text{1.NaNO}_2\text{/HCl}\ 0^\circ-5^\circ\text{C}} (\text{F})$$

[Assume that '**A**' to '**F**' are major organic products]
Now, identify the only INCORRECT statement regarding above sequence of reactions:
(A) Both (A) and (E) forms a foul smelling compound on reaction with $CHCl_3$ and KOH.
(B) (A) is basic, while (B) is amphoteric.
(C) One mole of compound (F) on heating with excess silver nitrate solution, forms two moles of white precipitate which readily dissolves in NH_3(aq) solution.
(D) Both (E) and (F) on reaction with Zn/NH_4Cl followed by reaction with Tollen's reagent forms silver mirror.

SECTION – A
(One or More than one correct type)

*This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

22. The correct statement(s) about group-I metals is/are
(A) Reactivity of group-I metals towards water decreases down the group.
(B) Among the group-I metals, 'Li' spontaneously reacts with N_2 to give a coloured crystalline compound.
(C) All group-I metal halides are high-melting colourless solids.
(D) KCl is more soluble in H_2O than NaCl at 100^oC.

23. 10 moles of liquid 'A' and 10 moles of liquid 'B' are mixed in a cylindrical vessel containing a massless and frictionless piston arrangement. Initially a pressure of 2 atm is maintained on the solution. Now, the piston is raised slowly and isothermally. Assuming 'A' and 'B' to be completely miscible and forming an ideal solution.
$p_A^o = 0.6$ atm, $p_B^o = 0.9$ atm (where p^o is vapour pressure of pure liquid), then choose the INCORRECT answers: (Given: $\sqrt{2425} = 49.25$; $\sqrt{3} = 1.73$; $\sqrt{2} = 1.41$)

(A) The pressure below which the evaporation of liquid solution will start is 1.75 atm.

(B) If $\left(\frac{1}{4}\right)^{th}$ of the total moles of liquid solution taken initially is converted to vapours, then moles of 'B' in the vapour phase will be 3.825.

(C) The pressure below which the evaporation of liquid solution will start is 0.75 atm.

(D) If half of the total moles of liquid solution taken initially is converted into vapours, then moles of 'A' in vapour phase, will be 2.5.

24. 2.0036 litre aqueous solution containing sufficient sodium salt of 2,3-dimethyl butan-1,4-dioic acid is electrolysed in an electrolytic cell using 9.65 mA current for 2×10^6 sec using inert Pt-electrodes with 100% current efficiency. Select the correct option(s): (Assume the density of water 1 g/ml, K_w of water at 25^oC is 1×10^{-14} and consider that OH^- ions occupy negligible volume)

(A) Mass of hydrocarbon formed at anode is 5.6 gm.

(B) pH of solution after electrolysis is 13 at 25^oC.

(C) Gas evolved at cathode occupies 2.24 litre volume at STP.

(D) Gas(es) evolved at cathode and anode occupies 8.96 litre volume at STP.

SECTION – B

(Numerical Answer Type)

*This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.*

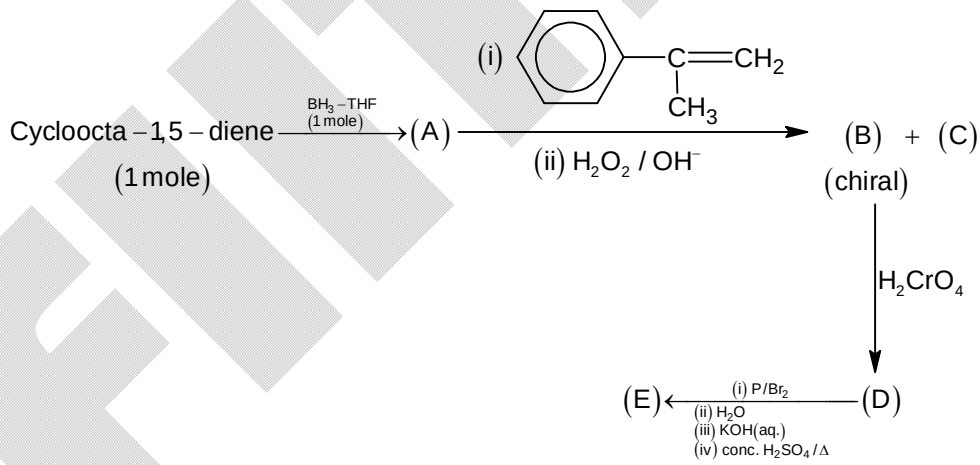
25. What is the sum of the squares of the magnetic moment (spin only) in the following complexes:

$\left[Ni(NH_3)_2Cl_2\right]$; $[RhF_6]^{3-}$; $\left[Cu(NH_3)_4\right]^{2+}$; $[PtCl_4]^{2-}$, $\left[Co(H_2O)_6\right]^{3+}$;

$\left[Ag(CN)_2\right]^-$; $[MnCl_4]^{2-}$; $\left[Fe(H_2O)_5(NO)\right]SO_4$

[Atomic number : Ni = 28; Rh = 45; Cu = 29; Pt = 78; Co = 27; Ag = 47; Mn = 25; Fe = 26]

26.



If the sum of all organic product(s) formed in the above sequence of reactions be 'x',

The sum of all organic product(s) formed in the above sequence of reactions which are having plane of symmetry be 'y', and

the sum of all organic product(s) formed in the above sequence of reactions which are having centre of symmetry be 'z',

then find the value of (x + y + z).

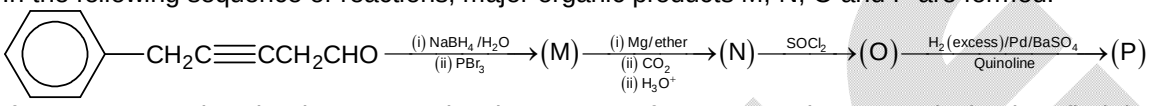
[here the term '**product(s)**' is referred for the major organic compounds '**A**' to '**E**' formed in the above sequence of reactions. Also, assume the cyclooctyl ring to be planar.]

27. For the reaction,

$$A(g) \underset{K_b = 2.4\ min^{-1}}{\overset{K_f = 1.80\ min^{-1}}{\rightleftharpoons}} B(g)$$

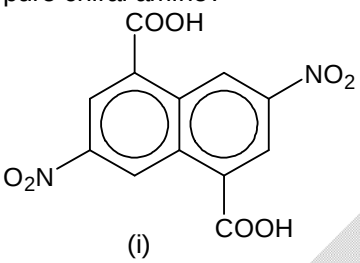
If time of 75% completion of 'A' is 'x' seconds. Where K_f and K_b are the rate constants of forward and backward reactions respectively, and reaction is started by taking only 1 mole of 'A' initially in a container of capacity 1 L.
In solid state, assume that 'A' crystallizes in bcc lattice then if the number of 4th nearest neighbours in bcc lattice be 'y', then what is the value of (x + y)?
$(\ell n 2 = 0.7)$

28. In the following sequence of reactions, major organic products M, N, O and P are formed.

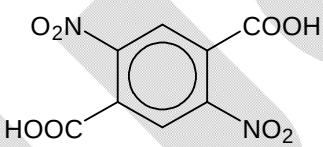


If M_1, M_2, M_3 and M_4 be the gram molecular masses of M, N, O and P respectively, then find the value of $(M_1 + 2M_3 - M_2 - M_4)$.
[Atomic masses: C = 12; H = 1; Br = 80; Cl = 35.5]

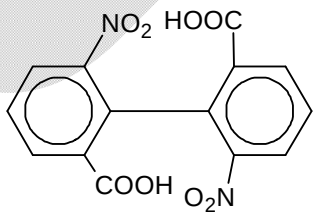
29. How many of the following carboxylic acid(s) can be resolved by reaction with an enantiomerically pure chiral amine?



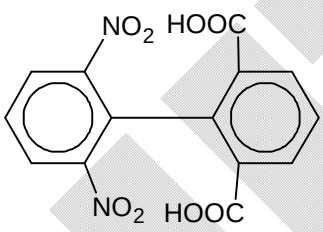
(i) ;



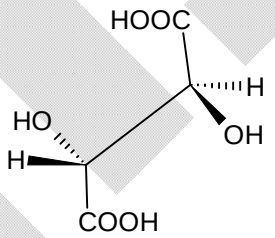
(ii) ;



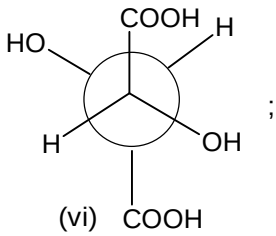
(iii) ;



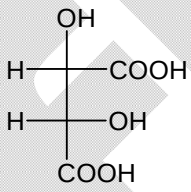
(iv) ;



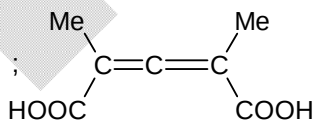
(v) ;



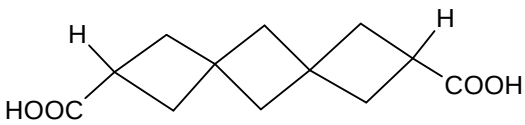
(vi) ;



(vii) ;



(viii) ;



(ix)

30. Some compounds/ions are given below:

$Si(CH_3)_4$; XeF_4; PCl_5; XeF_5^+; $B(OH)_3$; $\left[Ni(CN)_4\right]^{2-}$; SF_6; $\left[Zn(CN)_4\right]^{2-}$; $\left[IC\ell_4\right]^-$;

If the number of species in which at least five atoms may lie in same plane be 'x' and if the sum of planes of symmetry in $Si(CH_3)_4$ and SF_6 be 'y', then find the value of (x + y).

SECTION – C
(Numerical Answer Type)

This section contains **TWO (02) paragraphs.** Based on each paragraph, there are **TWO (02)** questions of numerical answer type. The answer to each question is a **NUMERICAL VALUE (XXXXX.XX).** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

Paragraph for Question Nos. 31 and 32

A reversible cyclic process involves 6 steps. In step-1 and 3, the system absorbs 500 J and 800 J of heat from a heat reservoir at temperature 250 K (T_1) and 200 K (T_3) respectively.
Step-2, 4 and 6 are adiabatic such that the temperature of one reservoir changes to that of next.
Total work done by the system in whole process is 700 J.

31. If total entropy change during steps-1,3 and 5 be 'x' J/K, then the value of $\left(\frac{x+4}{5}\right)$ is …..

32. If during step-5, the system exchanges heat from a reservoir at temperature 'T_5 K', then value of $\frac{T_1}{T_5}$ is:

Paragraph for Question Nos. 33 and 34

1 mole of an organic compound (X) on reductive ozonolysis consumes one mole of O_3 and produces two organic compounds (Y) and (Z). Compound (Z) on reaction with two moles of methanal in presence of NaOH produces 2,2-dimethylpropan-1,3 diol and another product (A).
(A) on acidification gives a compound which gives positive silver mirror test.
(Y) gives yellow precipitate with I_2 and alkali. (Y) on oxidation with SeO_2 gives (B) which on heating with conc. alkali followed by acidification, gives Mandelic acid as the only organic product.
Major product formed by the reaction of (Y) with ethyl acetate in presence of sodium ethoxide and ethanol on heating with acrolein (1 mole) in presence of excess alkali produces compound (C) as the major organic product.
While answering the following questions, follow the assumptions:
(i) Benzene ring (if any) does not undergo ozonolysis.
(ii) Consider all products (X, Y, Z, A, B and C) as the major organic products.
(iii) As per IUPAC system, Mandelic acid is named as '2-phenyl, 2-hydroxy ethanoic acid' and acrolein is named as prop-2-enal.
[Atomic masses: C = 12, O = 16, N = 14, H = 1, Na = 23, K = 39]

33. The ratio of the molar mass of 'X' to the total number of ozonides that 'X' can form, is…….

34. In the compounds, X, Y, Z, A, B and C.
If the sum of the number of pi-bonds in all of the above product = m and if degree of unsaturation of (C) be n, then find the value of $\frac{2m}{n}$.

Mathematics

PART – III

SECTION – A
(One Options Correct Type)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

35. The solution of differential equation $\frac{dy}{dx}=\frac{7\sin x\cos x+3(1-\cos y)\cos x}{7\sin y\cos y-3\sin x\sin y-7\sin y}$ is

(A) $\ln\left(\cos^2 y+\sin^2 x\right)=\frac{3}{7}\tan^{-1}\left(\frac{\cos y}{\sin x}\right)+c$

(B) $(7\cos y-3\sin x-7)^2+40\sin^2 x=c$

(C) $\ln\left(|\cos x-1|+|\sin x|\right)=\tan^{-1}\left(\frac{\cos y+1}{\sin x}\right)+c$

(D) none of these

36. Let $f(x)=\frac{(\tan x+\cot x)^2-(\tan x-\cot x)-2}{(\tan x+\cot x)^2+(\tan x-\cot x)-2}$, $\forall\ x\in R-\left\{\frac{n\pi}{2}, n\in I\right\}$, then

(A) $[f(x)]\in\{-1, 0, 1, 2, 3\}$, (where [.] represents greatest integer function)
(B) $[f(x)]\in\{1, 2\}$ (where [.] represents greatest integer function)
(C) if $a\le f(x)\le b$, then $14(a+b)$ is a perfect square of natural number
(D) if $a\le f(x)\le b$, then $14(a+b)$ is a perfect cube of natural number

37. Let $\int_0^1 x^r f(x)\,dx=0\ \forall\ r=1, 2, 3, ...,$ n, where f(x) is an n^{th} degree polynomial such that $f(0)=\left(\frac{1}{\sqrt{3}}\right)$, then $\frac{\int_0^1 f(x)\,dx}{\int_0^1 (f(x))^2\,dx}$ is equal to

(A) $\sqrt{3}$ (B) $\sqrt{2}$
(C) 1 (D) 2

38. $\vec{a}=(\tan x)\hat{i}+(\tan y)2\hat{j}+\hat{k}$ and $\vec{b}=(\tan x)\hat{i}-(\tan y)\hat{j}-\hat{k}$ are two non-zero perpendicular vectors.
If $\vec{p}=(\cos x)\hat{i}+(\sin x)\hat{j}+(\sin y)\hat{k}$ and $\vec{q}=(\cos x)\hat{i}-(\sin x)\hat{j}+(\sin y)\hat{k}$, then

(A) $\vec{p}$ and $\vec{q}$ are linearly dependent $\forall$ x, y $\in$ R
(B) $\vec{p}$ is perpendicular to vector $\vec{a}$
(C) $\vec{q}$ is perpendicular to vector $\vec{a}$
(D) $\vec{p}$ is perpendicular to vector $\vec{q}$

SECTION – A

(One or More than one correct type)

*This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

39. Let $f(x) = \frac{x^3}{3} + \frac{x^2}{2} + x + 2$ and $g'(x) = (x^2 - 9)(x^2 - 4x + 3)(x^2 - 3x + 2)(x^2 - 2x - 3)$, and $h(x) = f(g(x))\ \forall\ x \in R$. If n_1 ad n_2 be number of points of local minimum and local maximum respectively of h(x), then

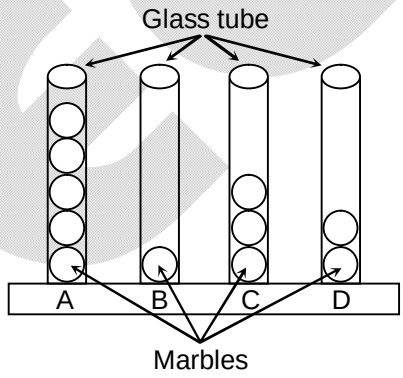
(A) $n_1 + n_2 = 7$

(B) $\frac{n_1}{n_2} \in N$

(C) $\frac{n_2}{n_1} \in N$

(D) $\frac{n_1^2 + n_2^2}{n_1 n_2} = 2$

40. The number of different design of show piece (as shown in figurer) that can be made using some or all the marbles from 12 number of difference coloured marbles is/are



(A) 11(13!) when exactly 10 marbles are used

(B) $\frac{5}{2}(14!)$ when all marbles are used

(C) $6\sum_{r=1}^{13} \frac{11!}{(13-r)!}(r^3 + 3r^2 + 2r)$

(D) $\sum_{r=0}^{12}\left(\frac{{}^{12}P_r}{4!}\right)$

41. If $A = \left[a_{ij}\right]_{n\times n}$ such that $a_{ij} = \begin{cases} 0 & ; \ i = j \\ -a_{ji} & ; \ i \neq j \end{cases}$ $\forall$ i, j = 1, 2, 3 and $A^2 + I = 0$, then which of the following statement is/are true?

(A) A is nilpotent matrix of order 3
(B) A is orthogonal matrix
(C) n can be 4 (where n is the order of the matrix A)
(D) n can be 5 (where n is the order of the matrix A)

SECTION – B

(Numerical Answer Type)

*This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.*

42. Let y = f(t) be the solution of differential equation $\frac{dy}{dt} + 2ty = t^2$, then $\lim_{t\to\infty}\left(\frac{12y}{t}\right)$ is

43. Let $\vec{a}, \vec{b}$ be the position vectors of points A and B respectively. Let C be mid-point of OB and OD is the angle bisector of $\angle AOB$, such that $\overrightarrow{AC}\cdot\overrightarrow{OD} = 0$. If $\angle AOB = \tan^{-1}\left(\frac{3}{4}\right)$, then $\left|\overrightarrow{OD}\right| : \left|\overrightarrow{AC}\right| = ?$

44. Let E_1, E_2, E_3 be three 2×2 matrices such that $E_iE_j + E_jE_i = 0$, $i \neq j$, i, j = 1, 2, 3. $E_i^2 = -I$, i = 1, 2, 3 where I is the identity matrix of order 2. If $A = xI + 2E_1 - \sqrt{2}E_2 + \sqrt{3}E_3$, where $x \in R$ such that $A(2xI - A) = (x^2 + n^2)I$, when $n \in N$, then n is

45. Let $3x + 4y + 7 = 0$ and $5x - 12y + 7 = 0$ be the pair of asymptotes to the hyperbola H. If the hyperbola passes through origin, then the eccentricity of hyperbola is $\frac{a}{b}$ (where HCF $(a^2, b) = 1$), then $\sqrt{a^2 - b}$ is

46. If a, b, c be three distinct integers that satisfy the equation $x^3 + ax^2 + bx + c = 0$, then $\left|a^3 + b^3 + c^3\right|$ is

47. Let origin lies inside the circle $x^2 + y^2 - 10x - 4\sqrt{2}y - c = 0$, $c > 0$. A PQ chord through origin is such that OP = 2, OQ = 8 (where O is the origin), then the radius of the circle is

SECTION – C
(Numerical Answer Type)

This section contains **TWO (02) paragraphs.** Based on each paragraph, there are **TWO (02)** questions of numerical answer type. The answer to each question is a **NUMERICAL VALUE (XXXXX.XX).** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

Paragraph for Question Nos. 48 and 49

a, b, c are distinct and $|a| + |b| = |c|$, also $|a + b\omega + c\omega^2|$ is minimum (where ω is the complex cube root of unity) . Now consider the lines having direction cosines connected by $al + bm + cn = 0$ and $l^2 + m^2 - n^2 = 0$

48. If $a \cdot b \cdot c = 0$, then angle between the lines is θ, then $\frac{1}{2}\tan^2\theta$ is

49. Total number of ordered triplets (a, b, c) are

Paragraph for Question Nos. 50 and 51

Let the line $\frac{x}{a} + \frac{y}{b} = 1$ intersect the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at two points A and B. If P be any variable point on the ellipse

50. The number of points P on ellipse is three, when the area of the $\Delta PAB = \frac{ab}{c}\left(\sqrt{d} - 1\right)$, then the value of $\left(\frac{d^2 + 1}{2c}\right)$ is equal to

51. The number of point P on ellipse is one, when the area of $\Delta PAB = \frac{ab}{c}\left(\sqrt{f} + 1\right)$, then the value of $\left(\frac{f + 1}{c}\right)$ is equal to